

Coxeter's rabbit

On p.13 of his "Introduction to Geometry", H.S.M. Coxeter invites the reader to see (and to use spontaneously) that with $s = (a+b+c)/2$, abc equals

$$(0) \quad s(s-b)(s-c) + s(s-c)(s-a) + s(s-a)(s-b) - (s-a)(s-b)(s-c)$$

Proof $s(s-b)(s-c) + s(s-c)(s-a)$

$$= \{ \text{algebra} \}$$

$$s(s-c)(2s-a-b)$$

$$= \{ \text{definition of } s \}$$

$$(1) \quad s(s-c)c$$

$$s(s-a)(s-b) - (s-a)(s-b)(s-c)$$

$$= \{ \text{algebra} \}$$

$$(2) \quad (s-a)(s-b)c$$

Because both expressions (1) and (2) contain a factor c , so does (0); for reasons of symmetry, (0) also contains factors a and b , i.e. is a multiple of abc . The coefficient equals 1 - as is trivially established with, say, $a, b, c := 2, 2, 2$ - and thus $abc = (0)$ has been proved. (End of Proof)

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